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Towards a theory of platform dynamics

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Abstract

I introduce a dynamic framework to analyze platforms. The (single) platform owner sets prices at the beginning of each period. Agents (buyers, sellers, readers, consumers, merchants, etc.) make platform membership decisions occasionally. I show that an optimal platform pricing addresses two externalities: across sides and across time periods. This results in optimal prices which depend on platform size in a nontrivial way. By means of numerical simulations, I examine the determinants of equilibrium platform size, showing that the stationary distribution of platform size may be bimodal, that is, with some probability the platform remains very low or takes very long to increase in size. I also contrast the dynamics of proprietary versus nonproprietary (i.e., zero-priced) platforms, and consider the specific case of asymmetric platforms (one side cares about the other but not vice versa).

KEYWORDS
dynamics, platforms, two-sided markets

1 | INTRODUCTION AND OVERVIEW

Market platforms are as old as human society. However, as the "digital revolution" unfolds, their number and economic importance grows considerably. Some platforms are household names (Alibaba, Airbnb, Uber, Apple's iOS, and Sony's PlayStation); many other are less known or have fallen into oblivion: For each successful platform, there are dozens of other platforms that "failed to launch."

This observation motivates much of my analysis, which addresses a number of issues related to two-sided platforms, both positive and normative issues: Why do some platforms succeed while others fail? To what extent is there a chicken-and-egg problem in platform formation (e.g., buyers do not want to join a trading platform unless there are sellers, and vice versa)? If so, should a platform owner subsidize membership? More generally, what is the optimal pricing policy for a platform owner? Is welfare higher or lower in a proprietary platform vis-à-vis a nonproprietary one (where prices are uniformly set a marginal cost level)? How does platform size evolve over time?

These and various other questions have been addressed in past research by a series of papers on two-sided markets. One common feature of most of these papers is that they are *static*: Both platform owners and agents make one-time decisions regarding prices, membership, and use. Static models have a long tradition in economics, and for a good reason, they provide a good approximation for equilibrium "once the dust settles," that is, once convergence to equilibrium takes place.

However, anecdotal evidence from many markets suggests that time plays an important role in agents' decisions. Specifically, inertia is an important force in many agents' decisions. Take smartphones, for example, while usage decisions are made on a regular basis, "membership" decisions are only made at irregular moments in time: you do not decide everyday which phone to own, much less which smartphone operating system to use. In other words, platform membership decisions are durable decisions.

In this paper, I propose a dynamic framework for the analysis of two-sided platforms. In each period, active agents—those who are called to make membership decisions—decide whether to join, stay in, or leave the platform, knowing the platform's current state (namely, the number of type a and type b agents) as well as the prices charged by the platform owner (prices which I assume depend on current platform size but are otherwise history independent). Agents are myopic and essentially solve a single-agent decision problem, namely utility maximization given prices and platform size.

This is very different from the static approach to platforms, when one assumes all agents make membership decisions *simultaneously* based on prices and on *expectations* about other agents decisions. In this context, coordination (the "chicken-and-egg" problem) becomes a major issue (in game theory terms, multiple equilibria are common). By contrast, my agents solve a series of single-agent decision problems based on prices and *actual* platform size (as opposed to *expectations* about platform size). This assumption, while somewhat extreme, provides a more realistic approximation to many real-world platforms comprising a large number of agents. In these real-world cases, coordinated equilibrium shifts seem rather unreasonable: It might well be a Nash equilibrium for the entire world to adopt Bitcoin tomorrow, but that does not make the model a realistic description of reality.

From an analysis point of view, one advantage of my dynamic approach is that there exists a unique equilibrium. However, this equilibrium induces a distribution of platform size at time t (as well as a stationary distribution) which is typically bimodal: With some probability platform size remains small or zero (failure to launch), and with some probability it takes off. This contrasts with static models, which typically admit multiple equilibria (e.g., one with successful platforms and one with unsuccessful platform).

The distinction between multiple equilibria (static model) and a unique equilibrium with multiple outcomes (dynamic model) is not merely semantic. My model's equilibrium distribution of platform size is continuous with respect to the relevant exogenous parameters. This allows me to perform meaningful comparative statics: How equilibrium outcomes change as exogenous parameters change. By contrast, equilibrium multiplicity in many static models makes comparative statics rather problematic.

In this paper, I explore a series of comparative dynamics exercises which illustrate the framework at work. In Section 4, I show how the probability of failure to launch (by time t or in the long run) depends on exogenous parameters such as the mean and standard deviation of agent valuations or the strength of platform effects. For example, the framework allows the analyst to answer questions such as: How does the likelihood the platform will have taken off after 1 year increase if the platform owner increases quality by 10% (as reflected in consumer valuations)?

In Section 5, I contrast proprietary and nonproprietary platforms, where by nonproprietary I mean a platform, such as Craigslist, that does not charge users. I provide sufficient conditions such that one or the other performs better from a social-welfare point of view. The advantage of nonproprietary platforms is that they minimize price-cost distortions (Harberger triangles). Against this, the advantage of proprietary platforms is that they may optimally set negative prices at some states, something that does not happen with nonproprietary platforms.

In Section 6, I consider the case of asymmetric platforms, those where side b cares more about side a than side a cares about side b. This presents interesting and complex optimal pricing challenges, well beyond what one would find in static models. For example, a proprietary platform first takes off on the b side and only later on do a types join in. Intuitively, the value of subsidizing a types is greater the greater the number of b types, which explains why the membership fee paid by a types may actually go down over time (as the number of b types increases). In a nonproprietary asymmetric platform, by contrast, none of these price patterns take place, which may imply that the platform never really takes off.

Before delving into comparative dynamics, in Section 2 I present the basic framework and in Section 3 I study the structure of optimal pricing in a dynamic context. I show that the optimal platform pricing addresses two externalities: across sides (Armstrong, 2006; Rochet & Tirole, 2003) and across time periods (Cabral, 2011). This results in optimal prices which depend on platform size in a nontrivial way. Specifically, I provide sufficient conditions (and intuition) for the price charged to agent i to be increasing in the number of j agents and decreasing in the number of i agents. Related to this, I show that under some conditions Weyl's (2010) "insulating prices" may be interpreted as a reduced form of the optimal Markov prices.

From a methodological point of view, in this paper I employ much of the "machinery" developed in Cabral (2011). There are, however, several important differences. Cabral (2011) deals with competition between two one-sided networks when agents have no outside option. By contrast, the present paper considers the user decision between a two-sided platform and an outside option of exogenously given value. This allows me to focus on a different set of issues. First, I study the shape of the optimal pricing function in the presence of two types of agents (Section 3). Second, I examine platform size dynamics, in particular the problem of "failure to launch" (Section 4). In fact, one interesting feature of my analysis is to show that failure to launch does not require platform competition or complicated consumer expectations.[1] My results combine numerical simulations and analytical results for low values of the discount factor. Elsewhere (Cabral, 2012) I argue that this is a powerful combination. Numerical simulations in isolation run the risk of burying interesting economics in a sea of numbers. Analytical results, in turn, are difficult to come by in highly nonlinear dynamic models. However, as shown by Budd, Harris, and Vickers (1993) and others, a great deal can be learned from linearizing the model around $\delta = 0$.

2 | MODEL

Consider a proprietary platform that brings together type a and type b agents. Time is discrete and runs indefinitely: $t = 1, 2, \ldots$ The total number of agents is given by η_i, $i = a, b$, that is, η_i is the potential number of type i agents who may join the platform. In each period, the *actual* number of agents who belong to the platform is given by x_i type i agents, $i = a, b$. We thus have $0 \le x_i \le \eta_i$.

The model's crucial feature is that, at random moments in time, each agent is called to reassess its decision regarding platform membership. In other words, I assume that platform membership is a "durable" decision. More specifically, each period starts with a platform of size $\mathbf{x} \equiv (x_a, x_b)$. At that moment, the platform owner sets prices (membership fees) $\mathbf{f}(\mathbf{x}) \equiv (\tilde{f}_a(\mathbf{x}), \tilde{f}_b(\mathbf{x}), f_a(\mathbf{x}), f_b(\mathbf{x}))$. The ~ symbol indicates fees to be paid by nonplatform members (i.e., I allow for the possibility of discrimination between new and existing platform members),[2] and the subscripts a, b denote the type of agent. I restrict the analysis to Markov strategies (where the state is given by $\mathbf{x}$). However, for simplicity I will omit the argument in many of the equations below.

After the platform owner sets prices, nature chooses a particular agent, whom I will designate as the "active" agent. With probability θ_i the active agent is of type i, where $\theta_a + \theta_b = 1$, and within a given type each agent becomes active with equal probability. In particular, both current platform members and nonmembers become active with equal probability. In other words, platform membership is a durable but not definitive decision: I allow for agents to leave the platform.

Agents are heterogeneous regarding the outside option, that is, the value of not joining the platform. Specifically, each agent's outside option is drawn from the cdf $\Phi_i(\cdot)$ (where $i = a$ or $i = b$, as the case may be). Moreover, a type i agent derives utility $\psi_i(\mathbf{x})$ from belonging to the platform. For the purposes of the present paper I assume that agents are "myopic," in the sense that they take the current payoff into account when making platform membership and platform usage decisions. Strictly speaking, this does not imply that agents do not consider future utility, rather that in their calculations they assume platform size remains fixed.[3]

In each period, the active agent may choose to stay on/leave/join/stay out of the platform, as the case may be. In other words, an active agent is an agent called to make a platform membership decision, not (necessarily) an agent who is currently a platform member. I denote by $q_i(\mathbf{f}, \mathbf{x})$ the probability that a type i active agent who is a current platform member chooses to remain in the platform, and by $\tilde{q}_i(\mathbf{f}, \mathbf{x})$ the probability that a type i active agent who is not a current platform member chooses to join the platform.

I denote the platform state before the active agent's decision by $\mathbf{x} \equiv (x_a, x_b)$, whereas the state after the active agent's decision is given by $\hat{\mathbf{x}}$. Note that $\hat{\mathbf{x}}$ may be equal to $\mathbf{x}$, but it may also be equal to $(x_a + 1, x_b)$, $(x_a - 1, x_b)$, $(x_a, x_b + 1)$, or $(x_a, x_b - 1)$. Specifically, I will use the notation $\mathbf{x}_i^+$ (resp., $\mathbf{x}_i^-$) to denote the vector that is obtained from $\mathbf{x}$ by adding (resp., subtracting) an agent of type i. Thus $\hat{\mathbf{x}} \in \{\mathbf{x}, \mathbf{x}_a^+, \mathbf{x}_a^-, \mathbf{x}_b^+, \mathbf{x}_b^-\}$.

Finally, period payoffs are paid: Nonplatform members receive their outside option; existing platform members, $\psi_i(\hat{\mathbf{x}})$, $i = a, b$; a new platform member, $-\tilde{f}_i(\mathbf{x}) + \psi_i(\hat{\mathbf{x}})$; a renewing platform member, $-f_i(\mathbf{x}) + \psi_i(\hat{\mathbf{x}})$; and the platform owner, $\tilde{f}_i(\mathbf{x})$, $f_i(\mathbf{x})$ or 0, depending on whether a membership fee is or is not charged (and what kind of platform membership fee is charged). In addition to current period payoffs, the platform owner expects a discounted value $\delta\, v(\hat{\mathbf{x}})$.

There are two sources of randomness in the model. One is that each Period 1 agent is selected by nature to be an active agent. This occurs with probability θ_i / η_i for a type i agent. Second, I assume an agent outside of the platform receives a random payoff ξ_{ikt} each period, a value which does not depend on the platform size, or the payoffs received by other agents, or the same agent in other periods. I assume ξ_{ikt} (observable by the agent at the beginning of the period) is distributed according to cdf $\Phi_i(\xi)$ satisfying the following assumption:

Assumption 1. $\Phi_i(\xi)/\phi_i(\xi)$ is increasing in ξ.

This is a standard assumption in auction theory and other areas of economics. It is satisfied by the uniform, normal, and other distributions.

The platform value function is given by

$$\begin{aligned} v(\mathbf{x}) \; = \; & \sum_{i=a,b} \left(\frac{\theta_i\,(\eta_i - x_i)}{\eta_i}\right) \tilde{q}_i(\mathbf{f}, \mathbf{x})\,(\tilde{f}_i\,(\mathbf{x}) + \delta\; v(\mathbf{x}_i^{+})) \\ & + \sum_{i=a,b} \left(\frac{\theta_i\, x_i}{\eta_i}\right) q_i(\mathbf{f}, \mathbf{x})\,(f_i\,(\mathbf{x}) + \delta\; v(\mathbf{x})) \\ & + \sum_{i=a,b} \left(\frac{\theta_i\,(\eta_i - x_i)}{\eta_i}\right)(1 - \tilde{q}_i(\mathbf{f}, \mathbf{x}))\;\delta\; v(\mathbf{x}) \\ & + \sum_{i=a,b} \left(\frac{\theta_i\, x_i}{\eta_i}\right)(1 - q_i(\mathbf{f}, \mathbf{x}))\;\delta\; v(\mathbf{x}_i^{-}). \end{aligned} \tag{1}$$

The first term on the right-hand side corresponds to the case when the active agent is type i and is currently not a platform member. This happens with probability $\theta_i\;(\eta_i - x_i)/\eta_i$. Moreover, I consider the possibility that this agent decides to join the platform. This happens with probability $\tilde{q}_i(\mathbf{f}, \mathbf{x})$. In this case, the platform makes a period profit of $\tilde{f}_i\,(\mathbf{x})$ and the value function starting next period is given by $v(\mathbf{x}_i^{+})$. In total there are eight possible cases (two types of agents, two possible membership status, and two possible membership decisions).

3 | PRICING

In this section, I derive a series of analytical results pertaining to optimal pricing, the proofs of which may be found in the appendix.

Proposition 1. *There exist $\delta' > 0$ such that, if $\delta \in (0, \delta')$, then $f_i(\mathbf{x})$ is increasing in x_j.*

The intuition for $f_i(\mathbf{x})$ to be increasing in x_j is that a type i willingness to pay is increasing in the number of members on the opposite side of the platform. This is a result already present in standard static two-sided markets: An increase in x_j leads to a nash equilibrium (NE) shift in platform demand by type i agents, and given minimal regularity conditions on the demand curve (in our case, Assumption 1), this leads to a higher price. The condition that $\delta < \delta'$ is required to make sure that dynamic considerations do not outweigh the (static) effect of a i-agent demand shift caused by an increase in x_j.

Proposition 2. *Suppose that $\psi_i(\mathbf{x}) = \overline{\psi}_i\, x_j$, with $\overline{\psi}_i \geq 0$, and* $\max_z z\; \Phi(\xi - z)$ *is a convex function of ξ. There exist $\delta' > 0$ such that, if $\delta \in (0, \delta')$, then $f_i(\mathbf{x})$ is decreasing in x_i.*

The intuition that $f_i(\mathbf{x})$ is increasing in x_j (Proposition 1) is fairly straightforward. By contrast, the intuition that $f_i(\mathbf{x})$ is decreasing in x_i is less straightforward. Basically, it can be shown that if $\psi_j(x_i)$ is linear and $\max_z z\; \Phi(x - z)$ is a convex function of x, then $v(\mathbf{x})$ is convex in x_i.[4] This implies that the dynamic premium from attracting a type i to the platform is *increasing* in platform size: The bigger the platform is, the more the platform owner has to gain from adding a new member.

Cabral (2011) provides similar convexity results. More generally, convexity of the value function is the driving force underlying the dynamics in models featuring learning-by-doing, innovation, or network effects (see, e.g., Budd et al., 1993; Cabral & Riordan, 1994). Proposition 2 follows along similar lines. Unlike Proposition 1, Proposition 2 is an inherently dynamic result. To the extent that agent is utility only depends on the number of j agents in the platform, we would expect optimal prices charged to i agents to be independent of the number (or expectation of the number) of i agents joining the platform. In a dynamic context, however, there is a future value of accepting a new i agent in the platform, and under some regularity conditions this value is increasing in x_i. Intuitively, along the equilibrium path both sides of the platform grow in tandem. Therefore, when x_i increases x_j also increases. Finally, a higher value of x_j implies that the value of attracting an additional i agent is greater, as it induces a positive externality on a greater number of agents.

Proposition 3. *Suppose that $\psi_i(\mathbf{0}) = \Phi_i(0) = 0$. There exist $\delta' > 0$ such that, if $\delta \in (0, \delta')$, then $\widetilde{f}_i(\mathbf{0}) < 0$.*

Proposition 3 states that dynamic considerations may imply that it is optimal to set negative prices. This is a well-known feature from dynamic pricing theory, including pricing with learning-by-doing (Cabral & Riordan, 1994) or with network effects (Cabral, Salant, & Woroch, 1999). In Section 5, we will see an important application of Proposition 3 to the comparative performance of proprietary and nonproprietary platforms (the latter defined by $\widetilde{f}(\mathbf{x}) = f(\mathbf{x}) = 0$).

3.1 | Static versus dynamic pricing

One of my goals is to compare the dynamic framework to various static frameworks previously proposed in the literature. Usually one thinks of static models as good approximate descriptions of steady-state equilibria of a dynamic process. Are these static models appropriate approximations? To address this question, it is useful to consider the one-shot game that best corresponds to the dynamic framework presented above.

Consider a model with a proprietary platform and η_i agents of type i, where $i = a, b$. In a first stage, the platform owner sets prices f_i. Following Weyl (2010), I consider the possibility of outcome contingent prices, that is, $f_i(x_j)$—what Weyl (2010) designates by "insulating price."[5] Note that $f_i(x_j)$ in the static model is different from $f_i(\mathbf{x})$ in the dynamic model. In one case, we have a mapping that is offered to consumers and, depending on the equilibrium value of x_j, a specific price is paid by type i consumers. By contrast, in the dynamic model, in each period active consumers pay a price $f_i(\mathbf{x})$ which is a function of the particular state $\mathbf{x}$ in that period.

Next, nature generates an outside option for each agent, each value drawn from cdf $\Phi_s(\cdot)$. Then all agents simultaneously decide whether to join the platform. Finally payoffs are received: Nonplatform members earn their outside option; a type i platform member gets $\psi_i(x_j) - f_i$; and the platform owner earns $\sum_{i=a,b} x_i f_i$, where x_i is the number of type i agents who join the platform.

It follows from the results in Weyl (2010) that equilibrium prices are given by $\widehat{f}_i(x_j) = \psi_i(x_j)$. Now let $\Phi^\alpha(x)$ be the trivial cdf corresponding to $x = \alpha$ with probability 1. In other words, $\Phi^\alpha(x)$ corresponds to the case when there is no agent heterogeneity: All agents' outside option is equal to α.[6] The next result shows that, in the limit case when the outside option is identical for all agents, the optimal prices in the dynamic game are identical to the insulating prices of the static game.

Proposition 4. *Suppose that $\psi_i(\mathbf{x}) = \psi_i(x_j)$. Then*

$$\lim_{\{\Phi_i(\xi)\}_t \to \Phi_i^0(\xi)} f_i(\mathbf{x}) = \widehat{f}_i(x_j).$$

In other words, Weyl's (2010) insulating prices $\widehat{f}$ correspond to the optimal prices in the dynamic game. Although there is an exact correspondence between equilibrium prices in the dynamic model and insulating prices in the static model, the nature of these prices is quite different. Insulating prices are a form of "insurance" to platform participants, namely insurance against nonparticipation by other participants. In equilibrium, platform members only pay one price, namely, $f_i(\widehat{x}_j)$, where $\widehat{x}_j$ is the equilibrium number of type j platform members.[7] In the dynamic game, by contrast, several of the prices $f_i(\mathbf{x})$ are observed along the equilibrium path. In other words, the platform does not "insure" agents, rather the platform "subsidizes" early adopters to compensate them for the low utility of joining the platform at that stage. In fact, to the extent that ψ_i is negative but close to zero whereas δ is positive, negative prices are observed along the equilibrium path (unlike the symmetric equilibrium in Weyl, 2010).[8]

4 | PLATFORM SIZE DYNAMICS

In the previous section, I analyzed optimal pricing by a proprietary platform, deriving (analytically) some typical patterns. A second point of interest in the study of platforms is the dynamics of platform size. For example, even if the

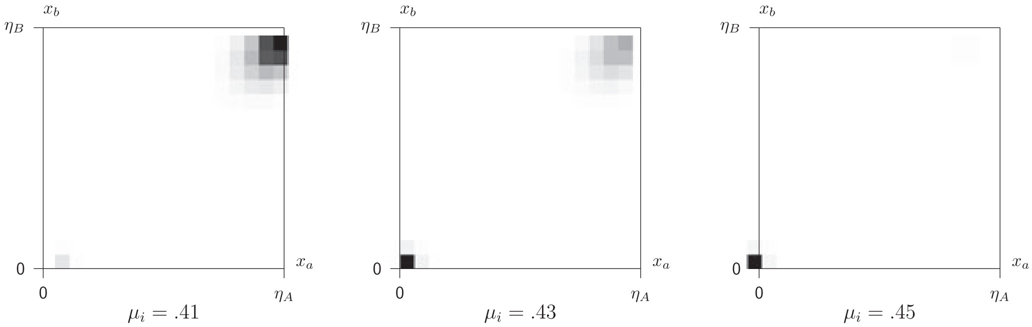

FIGURE 1 Stationary distribution of platform size for various values of μ. Other parameter values are $\delta = 0.9$, $\eta_i = 15$, $\theta_i = 0.5$, $\sigma_i = 0.2$, $\psi_i = 0.1$

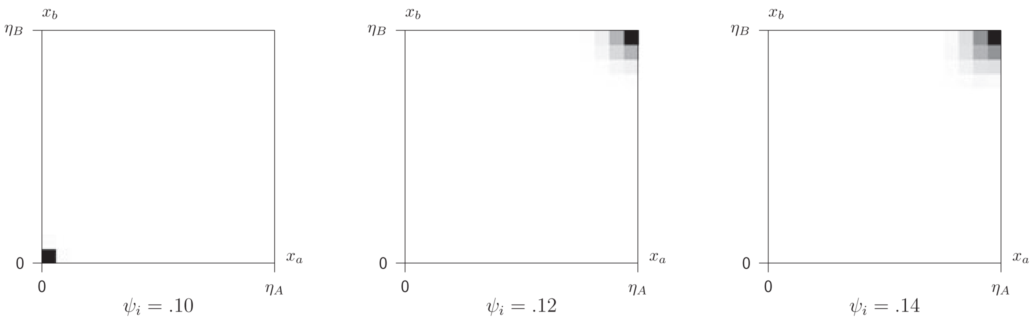

FIGURE 2 Stationary distribution of platform size for various values of ψ. Other parameter values are $\delta = 0.9$, $\eta_i = 15$, $\theta_i = 0.5$, $\mu_i = 0.5$, $\sigma_i = 0.2$

stationary distribution corresponds to a large platform membership, it may take a while for adoption to get started. Moreover, even if the stationary distribution of platform size is unimodal, the distribution after finite t may be bimodal. In this section I study the dynamics of platform size by means of numerical computations.[9] For the purpose of numerical computations, I assume that the outside option is normally distributed: $\xi \sim N(\mu, \sigma)$, whereas platform membership value for type i is linear in the number of type j agents: $\psi_i(\mathbf{x}) = \psi_i\, x_j$.

Figure 1 depicts the stationary distribution of platform sizes for different values of the average outside option. Specifically, the figure consists of a heat map where square darkness is proportional to density after 1 million periods (the figure is identical if I consider a longer horizon). As the figure shows, as platform membership becomes relatively less attractive (higher values of μ) the stationary distribution of platform sizes places more and more weight on $\mathbf{x} = 0$, that is, the probability of "failure to launch" is greater.

I mentioned earlier that an important feature of the (unique) equilibrium of the dynamic model is that the stationary distribution over states varies continuously with respect to the model's exogenous parameters. Figure 1 qualifies that statement by showing that, given bimodality of the stationarity distribution, even small changes in exogenous parameters may lead to substantial changes in the outcome. Specifically an increase in the average value of the outside option by 0.2 standard deviations moves the outcome from nearly guaranteed success to nearly guaranteed failure.

Figure 2 depicts a series of simulations which, together, show the effect of changing the degree of externalities. As one would expect, increasing the value of ψ implies a greater likelihood of the outcome whereby the platform takes off. In fact, similar to Figure 1, the outcome can be "discontinuous" with respect to the value of ψ, "jumping" from an equilibrium where almost all of the mass is at $\mathbf{x} = 0$ to one where most of the mass is at $\mathbf{x} = (\eta_a, \eta_b)$.

Figure 3 considers still another comparative-statics exercise, this time with respect to the value of σ, the standard deviation of the outside option. The results suggest that, starting from an equilibrium where the platform does not take off, an increase in σ leads to an increase in steady-state platform size. This is reminiscent of the idea that lead adopters, a concept first introduced by Rogers (1962) in the study of diffusion of innovations. In the present context, agents with a high valuation for the platform play the essential role of helping the platform take-off.

One can also find cases when an increase in σ leads to a shift in the stationary distribution from a bimodal distribution (as in the middle panel of Figure 1) to a unimodal distribution (as in the right panel of Figure 3). In static

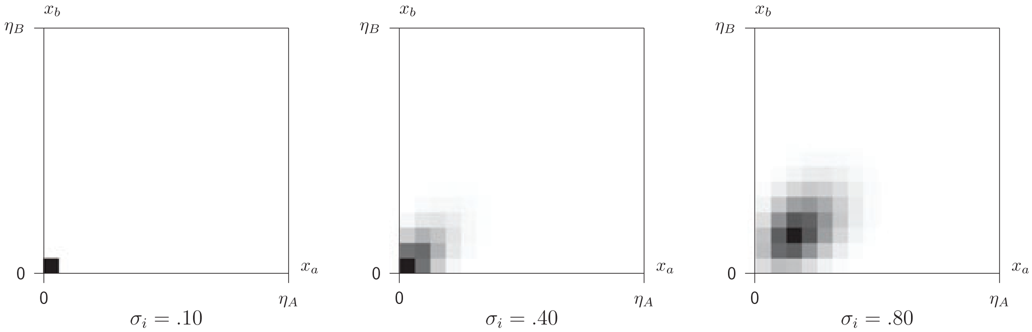

FIGURE 3 Stationary distribution of platform size for various values of σ. Other parameter values are $\delta = 0.9$, $\eta_i = 15$, $\theta_i = 0.5$, $\mu_i = 0.5$, $\psi_i = 0.1$

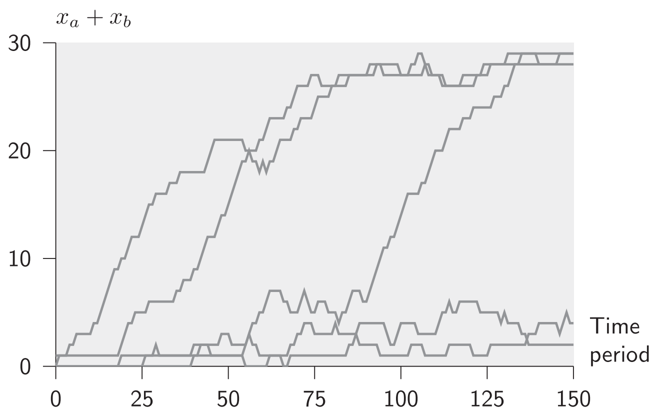

FIGURE 4 Sample paths starting at $\mathbf{x} = \mathbf{0}$. Parameter values are $\delta = 0.9$, $\eta_i = 15$, $\theta_i = 0.5$, $\mu_i = 0.4$, $\sigma_i = 0.3$, $\psi_i = 0.2$

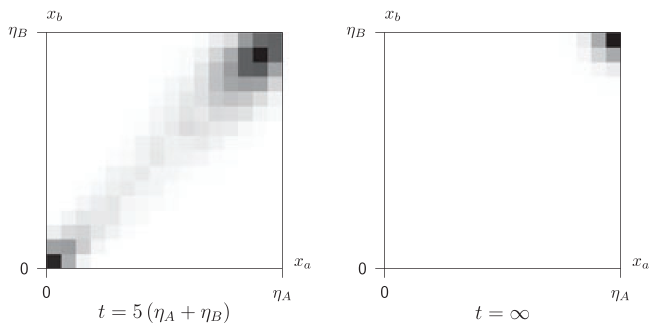

FIGURE 5 Evolution of $d_t(\mathbf{x})$ starting from $\mathbf{x} = 0$ Parameter values are $\delta = 0.9$, $\eta_i = 15$, $\theta_i = 0.5$, $\mu_i = 0.4$, $\sigma_i = 0.3$, $\psi_i = 0.2$

models, one possible "solution" to the chicken-and-egg problem is simply to assume that the degree of agent heterogeneity is sufficiently high (Rochet & Tirole, 2003). Figure 3 provides the dynamic counterpart of this idea.

As Keynes aptly put it, "in the long run we are all dead." In the present context, this begs the question of what the actual paths of platform size are, in addition to the steady-state distribution of platform sizes. Figures 4 and 5 address this question in two ways. Figure 4 depicts a series of time paths for platform size. For simplicity, I plot the combined number of type a and type b agents. Since the model is symmetric, the patterns would be similar if we were to consider only the number of type a or type b agents. The striking feature of these paths is that the value of $x_a + x_b$ either remains very small or increases to values close to $\eta_a + \eta_b$.

Figure 5 is the natural counterpart to Figure 4. It shows the stationary distribution of platform sizes at $t = 5\ (\eta_A + \eta_B)$ (i.e., when each agent has made an average of five decisions), as well as the stationary decision. The message is clear: Even though the stationary distribution corresponds to near full adoption—that is, $\mathbf{x} \approx (\eta_a, \eta_b)$—it takes "very long" to get there. Naturally "very long" is a matter of judgment and of parameter values. Nevertheless, it is interesting to see how after 150 periods there is still a significant probability that the platform has not taken off.

It is also interesting to see that the distribution at t is bimodal and has most of its mass along the 45° line, suggesting that platforms grow when both sizes grow together. Note, however, that I have been assuming that both network effects and outside option are symmetric, which is a strong simplifying assumption. In Section 6, I will consider asymmetric case and show that asymmetric parameter values naturally lead to asymmetric outcomes in terms of platform size.

5 | PROPRIETARY VERSUS NONPROPRIETARY PLATFORMS

One important dimension of variation across platforms is ownership: eBay started out as a not-for-profit but later became a for-profit platform; Craig's list is still not-for-profit. This suggests an interesting research question: What difference does platform status make in terms of platform size? Define nonproprietary platform as $f_i(\mathbf{x}) = 0$, whereas a proprietary platform corresponds to the case considered in the previous sections.

My first analytical result is then that a proprietary platform increases the likelihood of a high-membership outcome.

Proposition 5. *Suppose that* $\Phi_i(\xi) = \Phi^\alpha(\xi)$, $\psi_i(\mathbf{0}) < \alpha$, $\psi_i(\eta_a, \eta_b) > 0$ *and* $\psi_i(\mathbf{x}) < \psi_i(\eta_a, \eta_b)$ *if* $\mathbf{x} \neq (\eta_a, \eta_b)$. *There exists a* $\delta' \in (0, 1)$ *such that, if* $\delta > \delta'$,

(a) *If the platform is nonproprietary, then platform size is always zero.*
(b) *If the platform is proprietary, then platform size converges to maximum size.*

Intuitively, the idea is that, in a proprietary platform, prices may be negative, whereas in a nonproprietary platform $\mathbf{f} = 0$ by definition. Note that negative price does not need to be interpreted literally: It could be price below cost (which I assume to be zero but may not be zero), or it could mean higher quality levels, for example, through the provision of content by the platform owner.[10]

My second analytical result provides a counterpart to Proposition 5. It states that, conditional on a platform taking off, a nonproprietary platform is superior from a social-welfare point of view.

Proposition 6. *Suppose that* $\Phi_i(0) = 1$ *and* $\phi_i(0) > 0$ *(*$i = a, b$*), that is, the outside option is (weakly) negative for all agents. There exists a* $\delta' > 0$ *such that, if* $\delta < \delta'$,

(a) *If the platform is nonproprietary, then eventually all agents join the platform.*
(b) *If the platform is proprietary, then in the steady-state platform size is lower than maximum size with strictly positive probability*

The intuition for Proposition 6 is as simple as the distortion resulting from monopoly pricing: There are agents whose relative valuation for platform membership is positive (negative outside option) who fail to join the platform on account of positive prices.

The top panels of Figure 6 illustrate the idea of Proposition 5 in a less extreme case then $\Phi_i(\xi) = \Phi_i^\alpha(\xi)$. As can be seen, platform size remains at zero (with high probability) when the platform is not proprietary but converges to near the maximum (with high probability) when the platform is proprietary. The key difference is that, under a proprietary platform, optimal prices when $\mathbf{x} \approx (0, 0)$ are strictly negative, whereas under a nonproprietary platform they are always zero (by definition).

By contrast, the bottom panels of Figure 6 illustrate Proposition 6. As can be seen, the proprietary platform induces a stationary distribution over platform sizes (left panel) that is dominated by the corresponding distribution of a nonproprietary platform (right panel). The idea is that strictly positive prices set by a proprietary platform lead agents to shy away from joining the platform, whereas no such positive prices are observed under a nonproprietary platform.

6 | ASYMMETRIC PLATFORMS

So far I have considered cases when $\psi_a(\mathbf{x}) = \psi_b(\mathbf{x})$. Suppose however that most of the platform externalities are one way; for example, type b values the presence of type a to a high degree but not vice versa. Figure 7 depicts the stationary

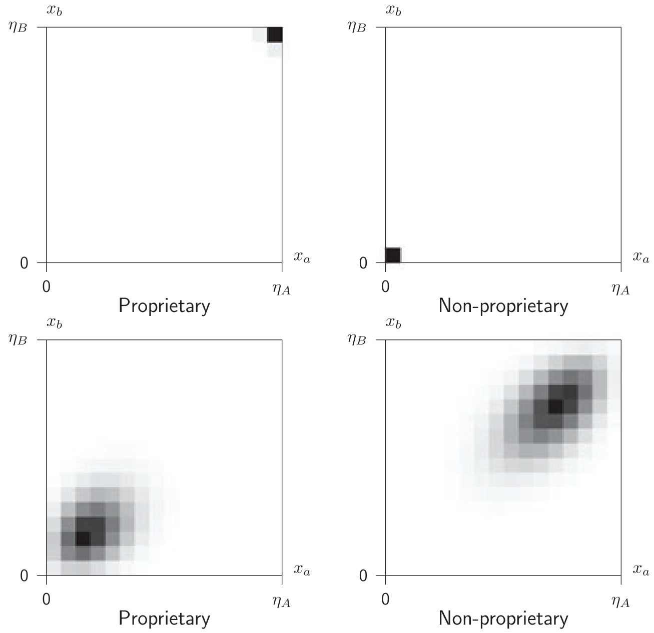

FIGURE 6 Proprietary versus nonproprietary platforms. Parameter values are $\mu_i = 1$, $\sigma_i = 0.2$, $\psi_i = 0.5$ (top panels) and $\mu_i = 0.5$, $\sigma_i = 1$, $\psi_i = 0.1$ (bottom panels)

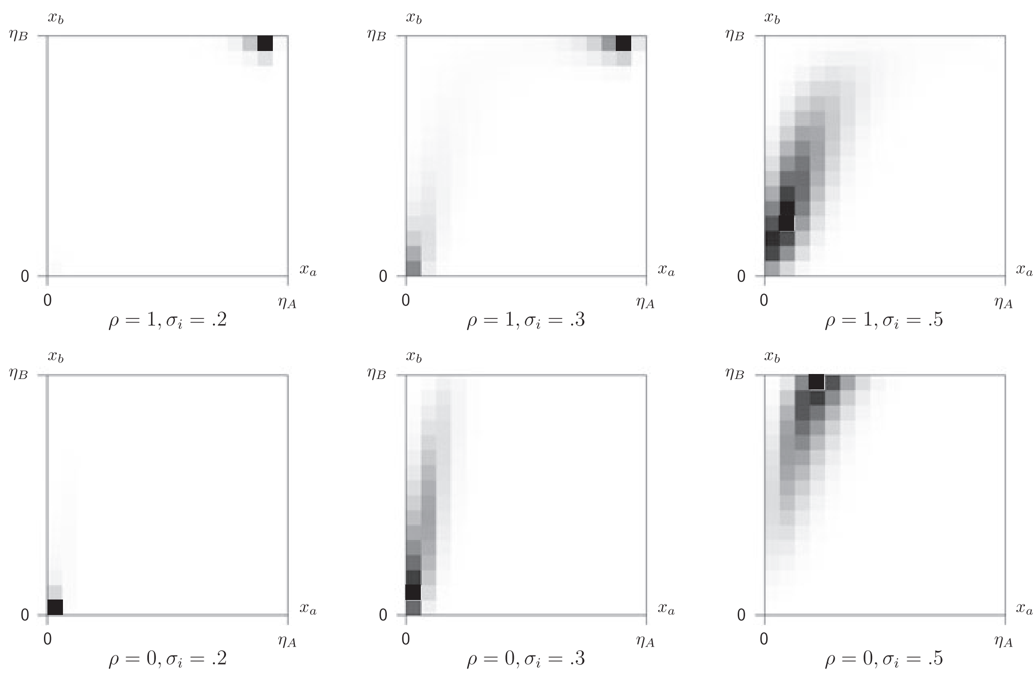

FIGURE 7 Stationary distribution of $\mathbf{x}$ with asymmetric platforms: $\psi_A = 0$, $\psi_B = 0.5$. Other parameter values are $\delta = 0.9$, $\eta_i = 15$, $\theta_i = 0.5$

distribution of platform sizes for three values of σ (the standard deviation of the outside option) and for the two platform ownership possibilities: proprietary ($\rho = 1$) and nonproprietary ($\rho = 0$).

The extreme values of σ (0.2 and 0.5) approximately reproduce the outcomes in Figure 6. The intermediate case ($\sigma = 0.3$) presents an intriguing pattern: Whereas the nonproprietary platform ($\rho = 0$, bottom center panel) shows a unimodal distribution, the proprietary platform ($\rho = 1$, top center panel) shows a bimodal distribution. In part, the intuition for the difference between $\rho = 0$ and $\rho = 1$ goes back to Proposition 5: To the extent that a proprietary platform is able to subsidize agents, it may be possible to achieve levels of platform take-off that would not be reached in a nonproprietary platform.

But there is more, the top central panel suggests that, on average, the path toward nearly full platform membership is rather asymmetric: Initially, membership originates primarily in b types, and only later—that is, once a significant number of b types have joined the platform—do a types join the platform as well.

The intuition for this asymmetric pattern is that a low ψ_a and a high ψ_b together imply that $f_a(\mathbf{x})$ is *decreasing* in x_b. This corresponds to a sort of extensive margin effect: Since type a agents produce a strong externality on type b agents, when many type b agents are present in the market the value created by subsidizing the a side is greater. This pattern corresponds to a state-dependent version of Armstrong's cross-platform subsidization result.

The property that $f_a(\mathbf{x})$ is decreasing in x_b is in stark contrast with Proposition 1, which provides sufficient conditions such that $f_a(\mathbf{x})$ is increasing in x_b.[11] How can the two results be reconciled? There are essentially two effects to consider. First, the direct demand effect, which is characterized by Proposition 1. If type a cares about type b, then everything else equals an increase in the number of type b agents is met by an increase in the fee charge to type a agents. To guarantee this is the only effect in play, Proposition 1 is based on the assumption that δ is infinitesimally small, so that dynamic effects are of second order with respect to the direct demand effect.

Second, there is a cross-subsidy or externality effect: To the extent that type b cares about type a, bringing an extra type a on board creates value on the b side of the market. The more type b platform members there are, the more value is created by a new a platform member. Everything else equal, this leads the platform to lower the fee it charges a agents. To guarantee that this is the only effect in play, Figure 6 is based on the assumption that $\psi_a(\mathbf{x}) = 0$, which effectively shuts off the first effect.

7 | CONCLUSION

The vast majority of the economic analysis of platforms assumes that, in equilibrium, platforms exist and attract a number of agents from both "sides." However, the history of platforms shows that "failure to launch" is a distinct possibility. In this paper, I use a dynamic model to look at the development of platform size. I show that, for a wide set of parameter values, platform size at time t follows a bimodal distribution: Either the platform achieves a "large" size or remains at very "low" size.

I show that optimal platform pricing addresses two externalities: across sides and across time periods. This results in optimal prices which depend on platform size in a nontrivial way. Specifically, I provide sufficient conditions (and intuition) for the price charged to agent i to be increasing in the number of j agents *and* decreasing in the number of i agents.

Among several other results, the numerical solution of the model for a variety of parameter values suggests that equilibrium platform size is greater for nonproprietary platforms, but the probability of successful platform launch may be greater under proprietary platforms.

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ENDNOTES

1 Other problems considered in the present paper but not in Cabral (2011) are the contrast between proprietary and nonproprietary platforms (Section 5) and asymmetric platforms (Section 6).

2 The assumption of price discrimination between new and old platform members is consistent with the practice of many platforms. In some cases, a different membership price is charged to new members. In other cases, joining the platform (e.g., Apple IOS or PlayStation) requires an investment in a durable good, which in turn provides a natural vehicle for differential treatment. From an analytical point of view, the discrimination case is easier to deal with.

3 One advantage of considering the myopic agent case is to make it clear that failure to launch is not necessarily the result of unfavorable agent expectations (as often is considered in static models). See Cabral (2011) for the analysis of forward-looking agents and the equilibrium comparisons with respect to myopic (or naïve) agents.

[4] I note that if ξ is normally or uniformly distributed, then the assumption that $\max_z z\,\Phi(x-z)$ is a convex function of x holds.

[5] Caillaud and Jullien (2003) also examine various possible contracting structures, including prices that are contingent on outcomes.

[6] Technically, and to be consistent with Assumption 1, I consider the limit case as all of the mass of $\Phi(x)$ is concentrated around α. Specifically, I consider sequences $\{\Phi_i(\xi)\}_t$ converging to $\Phi_i^\alpha(\xi)$ such that $\lim_{x\to\alpha^-}\Phi_i^\alpha(\xi)=0$ and $\lim_{\xi\to\alpha^+}\Phi_i^\alpha(\xi)=1$.

[7] Given symmetry, in a pure-strategy equilibrium $x_j=0$ or $x_j=\eta_j$.

[8] Veiga, Weyl, and White (2017) provide an informal description of Proposition 4: "One way to interpret this strategy is in dynamic terms: the platform charges a low price (or even offers a subsidy) early on, when there are few users, gradually raising the price as the platform develops."

[9] For details regarding the numerical solution of the model, see Cabral (2011).

[10] Caillaud and Jullien (2003), for example, state that "a negative price can be the consequence of gifts given to joining members, or the result of the addition of free services to the basic free-of-charge matching service" (p. 312).

[11] The fact that a simple change in parameter values leads to such a drastic qualitative change in the nature of optimal pricing suggests that experimentation may play an important role in platform optimization (see Peitz, Rady, & Trepper, 2017).

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APPENDIX A

Proof of Proposition 1. The membership probability functions is given by

$$\begin{aligned} q(\mathbf{f},\mathbf{x}) &= \Phi(\psi_i(\mathbf{x})-f_i(\mathbf{x})),\\ \tilde{q}(\tilde{\mathbf{f}},\mathbf{x}) &= \Phi(\psi_i(\mathbf{x})-\tilde{f}_i(\mathbf{x})). \end{aligned} \tag{A1}$$

The first-order conditions from maximizing $v(\mathbf{x})$ with respect to $f_i(\mathbf{x})$ and $\tilde{f}_i(\mathbf{x})$ are given by

$$\begin{aligned} q_i(\mathbf{f},\mathbf{x}) &+ \frac{d_{q_i}(\mathbf{f},\mathbf{x})}{d\,f_i(\mathbf{x})}\,(f_i(\mathbf{x})+\delta\,v(\mathbf{x})-\delta\,v(\mathbf{x}_i^-))=0,\\ \widetilde{q}_i(\mathbf{f},\mathbf{x}) &+ \frac{d_{\widetilde{q}_i}(\mathbf{f},\mathbf{x})}{d\,\widetilde{f}_i(\mathbf{x})}\,(\widetilde{f}_i(\mathbf{x})+\delta\,v(\mathbf{x}_i^+)-\delta\,v(\mathbf{x}))=0. \end{aligned} \tag{A2}$$

Define

$$\begin{aligned} H_i(z) &\equiv \frac{\Phi_i(z)}{\phi_i(z)},\\ \Gamma_i(z) &\equiv \frac{\Phi_i(z)^2}{\phi_i(z)} = H_i(z)\,\Phi_i(z). \end{aligned} \tag{A3}$$

Equations (A1) and (A2) imply

$$\begin{aligned} f_i(\mathbf{x}) &= H(\psi_i(\mathbf{x})-f_i(\mathbf{x}))-\delta\,v(\mathbf{x})+\delta\,v(\mathbf{x}_i^-),\\ \widetilde{f}_i(\mathbf{x}) &= H(\psi_i(\mathbf{x})-\widetilde{f}_i(\mathbf{x}))-\delta\,v(\mathbf{x}_i^+)+\delta\,v(\mathbf{x}). \end{aligned} \tag{A4}$$

Assumption 1 implies that, holding the value functions constant, f and $\widetilde{f}$ are strictly increasing in $\psi_i(\mathbf{x})$. Since ψ_i is increasing in x_j, it follows that at $\delta = 0$, $f_i(\mathbf{x})$ and $\widetilde{f}_i(\mathbf{x})$ are strictly increasing in x_j. By a continuity argument, $f_i(\mathbf{x})$ and $\widetilde{f}_i(\mathbf{x})$ are also strictly increasing in x_j for δ close to zero. □

Proof of Proposition 2. Substituting (A4) for $f_i(\mathbf{x})$ in (1), and simplifying, we get

$$\begin{aligned} v(\mathbf{x}) &= \sum_{i=A,B} \theta_i\left(\frac{\eta_i-x_i}{\eta_i}\right)(\Gamma_i(\psi_i(\mathbf{x})-\widetilde{f}_i(\mathbf{x}))+\delta\,v(\mathbf{x}))\\ &\quad+\theta_i\left(\frac{x_i}{\eta_i}\right)(\Gamma_i(\psi_i(\mathbf{x})-f_i(\mathbf{x}))+\delta\,v(\mathbf{x}_i^-)). \end{aligned} \tag{A5}$$

Define, for a generic variable z,

$$\mathring{z} \equiv z|_{\delta=0} \quad \dot{z} \equiv \left.\frac{d\,z}{d\,\delta}\right|_{\delta=0}.$$

If $H(0) = 0$ and $\psi_i(x_i, 0) = 0$, then $\mathring{v}(\mathbf{0}) = 0$ and $\mathring{v}(\mathbf{0}_i^+) > 0$. It follows from (A4) that $\widetilde{f}_i(\mathbf{0}) < 0$. Finally, suppose that $\psi_i(\mathbf{x}) = \overline{\psi}_i\,x_j$ (with $\overline{\psi}_i \geq 0$). This implies that $\mathring{v}(\mathbf{x})$

$$\begin{aligned} \mathring{v}(\mathbf{x})-\mathring{v}(\mathbf{x}_i^-) &= \theta_j\left(\frac{\eta_j-x_j}{\eta_j}\right)(\Gamma_j(\psi_j(\mathbf{x})-\mathring{\widetilde{f}}_j(\mathbf{x}))-\Gamma_j(\psi_j(\mathbf{x}_i^-)-\mathring{\widetilde{f}}_j(\mathbf{x}_i^-)))\\ &\quad+\theta_j\left(\frac{x_j}{\eta_j}\right)(\Gamma_j(\psi_j(\mathbf{x})-\mathring{f}_j(\mathbf{x}))-\Gamma_j(\psi_j(\mathbf{x}_i^-)-\mathring{f}_j(\mathbf{x}_i^-))). \end{aligned}$$

If $\max_z z\,\Phi(y-z)$ is a convex function of y, then both terms on the right-hand side of the above equation are increasing in i. It follows that $\mathring{v}(\mathbf{x})-\mathring{v}(\mathbf{x}_i^-)$ is increasing in i as well. From (A4) we get

$$\mathring{f}_i(\mathbf{x}_i^+)-\mathring{f}_i(\mathbf{x}) = H(\psi_i(\mathbf{x}_i^+)-\mathring{f}_i(\mathbf{x}_i^+))-H(\psi_i(\mathbf{x})-\mathring{f}_i(\mathbf{x}))=0,$$

since $\psi_i(\mathbf{x}_i^+) = \psi_i(\mathbf{x})$. Moreover,

$$\operatorname{sgn}(\dot{f}_i(\mathbf{x}_i^+)-\dot{f}_i(\mathbf{x})) = -\operatorname{sgn}((\mathring{v}(\mathbf{x}_i^+)-\mathring{v}(\mathbf{x}))-(\mathring{v}(\mathbf{x})-\mathring{v}(\mathbf{x}_i^-))).$$

Since $\mathring{v}(\mathbf{x})-\mathring{v}(\mathbf{x}_i^-)$ is increasing in i, we conclude that, in the neighborhood of $\delta = 0$, $f_i(\mathbf{x})$ is decreasing in x_i. □

Proof of Proposition 3. If $\tilde{f}_i(0) \geq 0$, then no platform membership ever takes place, thus $\mathbf{x} = 0$ in very period. Suppose that $\tilde{f}_i(0) = -\epsilon < 0$, where ϵ is infinitesimally small. In any given period, the cost of setting such a negative fee is of order ϵ^2: The probability of a membership decision is of order ϵ and the loss is ϵ. The gain, however, is of order ϵ: The probability of a membership decision is of order ϵ and the gain conditional on a membership decision taking place is positive and of first order of magnitude. □

Proof of Proposition 4. Consider first the dynamic model. Suppose that the outside option is equal to zero and consider the dynamic problem faced by a platform owner in state $\mathbf{x}$. If price is lower than $\psi_i(x_j)$, then an active i agent will join the platform (or remain in it, as the case may be). Since the platform's value function is increasing in the number of participant (no negative externalities), it is not optimal to set a price greater than $\psi_i(x_j)$. Since the agent's decision is the same for any price below $\psi_i(x_j)$, it is not optimal to set any price strictly lower than $\psi_i(x_j)$. Together, these results imply that $f_i(\mathbf{x}) = \tilde{f}_i(\mathbf{x}) = \psi_i(x_j)$. It follows from the results in Weyl (2010) that equilibrium prices in the static game are given by $\hat{f}_i(x_j) = \psi_i(x_j)$. Finally, the result follows by continuity of the distribution functions and the fact that the platform's optimal policy is based on strict inequalities. □

Proof of Proposition 5. Consider first part (a) of the result. By definition, a nonproprietary platform sets $f(\mathbf{x}) = \tilde{f}(\mathbf{x}) = 0$ for all $\mathbf{x}$. Beginning at $\mathbf{x} = (0, 0)$, this implies no (myopic) agent will ever join the platform: The benefit, $\psi_i(\mathbf{0})$, is lower than the outside option, α.

Consider now a proprietary platform and the following pricing strategy: $f_i(\eta_a, \eta_b) = \psi_i(\eta_j)$, and $f_i(\mathbf{x}) = \tilde{f}_i(\mathbf{x}) = \alpha$ for $\mathbf{x} \neq (\eta_a, \eta_b)$. With these prices agents always prefer to join (or remain in) the platform. If δ is sufficiently close to 1, then any pricing strategy leading to a stationary platform size different from $\mathbf{x}$ leads to a strictly lower discounted value. □

Proof of Proposition 6. Consider first the case of a nonproprietary platform. Since $\tilde{f}(\mathbf{x}) = f(\mathbf{x}) = 0$ and all outside options are negative, all agents join the platform. Consider now the case of a proprietary platform. Since all outside options are strictly negative almost surely, it is optimal to charge strictly positive fees at all times. Since the density of the outside option distribution is strictly positive at zero, an active agent chooses to leave (or not join) the platform with strictly positive probability. □